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Multi-Period and Multi-Objective Stock Portfolio Modeling Considering Cone Constraints

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Abstract

Nowadays, one of the major concerns of investors is choosing a realistic stock portfolio and making proper decisions according to an individual's utility level. It is essential to consider two conflicting goals of return and risk for profitability; as a result, balancing the above goals has been identified as an investment concern. This paper modifies and optimizes a multi-objective and multi-period stock portfolio considering cone constraints and uncertain and stochastic discrete decisions. Non-dominated Sorting Genetic Algorithm-II (NSGA-II) was used to solve the model due to the issue's complexity. Two objective functions in the model could be explained by maximizing expected returns and minimizing investment risk. The Pareto chart of the problem was drawn, which allows investors to make decisions based on various levels of risk. Another result obtained in this study is calculating the percentage of optimal amounts assigned to each asset, providing a base for investors to avert investing in unsuitable assets and incurring losses. Finally, a sensitivity analysis of essential parameters was performed, which is critical in this issue. According to the results, increasing the number of problem constraints provides a base for the model reaction, and the optimal percentage allocated to each asset varies. Therefore, this prioritizes restrictions in different situations and according to the investors' choice.

Keywords: Genetic algorithm, Optimization, Stock portfolio, Cone constraints, Multi-objective modeling, Discrete decisions.

1 | Introduction

According to Markowitz [1], the issue of selecting a stock portfolio could be considered a classic one consisting of two essential sections: risk and return. The main objective is either to maximize the expected return at a particular risk level or to minimize the chance at a specified level of return. Nowadays, many

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factors concerning investors, including selecting the optimal stock portfolio investment, assigning capital to corporate assets, and selecting a stock portfolio, contrast profitability and optimal risk due to the complex financial markets. Apart from this, efficient stocks to accomplish greater returns and less risk have become a challenging issue in financial management with the rising development of financial markets and the accessibility of various stocks in these markets.

Recently, Mehlawat et al. [2] extended fuzzy numbers to coherent fuzzy numbers for modeling an individual's insight into the stock market: optimistic, pessimistic, or neutral and asset returns. In their study, both multi-objective portfolio optimization and multi-period models were formulated. Conditional Value-at-Risk (CVaR) and mean absolute semi-deviation were considered risk measures, respectively.

Dai and Qin [3] focused on optimizing a multi-period portfolio problem under uncertainty considering minimum transaction costs. They assumed that the total available wealth is exponentially at the end of the investment horizon. An extra clarified framework of the total wealth was provided, which may make the operation of experimental calculation concise since it was a linear function of decision variables. Alongside this, the dynamic risk was considered during the investment horizon. Considering these sensible constraints derived from the complicated financial markets, they built a multi-period mean-VaR (value-at-risk) model to maximize the terminal wealth under the risk control over the entire investment. The genetic algorithm was used to solve the defined model, and two numeric examples were given to represent the performance of the proposed perception.

Khandan et al. [4] have introduced a different approach for multi-period investment of stock portfolio optimization based on the size of general fuzzy and used the scenario tree to cope with uncertainties. It also provides a base for investors to impose their tastes. It is unnecessary to model in the form of credit, obligation, and possibility by changing the optimistic-pessimistic parameters and considering all of the above constraints. The Epsilon-constraint method solved the proposed model to perform a single objective.

According to previous research, experts have considered various methods, including the fuzzy approach, considering different risks, and in some models, multi-objective functions and others in periodic multi-period of time. However, multi-objective and multi-period modeling simultaneously is a subject that is rarely discussed due to the various constraints and high complexity. Thus, our primary purpose is to discover a multi-objective and multi-period model so the investor can concurrently consider the two conflicting objectives of profitability and risk by choosing a model that could be utilized based on the level of the investor's utility. Investors will likely utilize a multi-period stock portfolio investment to reconsider allocating their capital from period to period. In the proposed model, the sensitivity analysis of applied essential restrictions provides a base for investors to increase their stock portfolio profitability by choosing stock portfolios, making decisions besides the two main objectives listed, and prioritizing existing restrictions. Since the return on capital during various periods cannot be considered steady, stock returns should be analyzed using fuzzy, robust, or probabilistic planning according to many choices available in the capital markets. Hence, the contributions of this research are a simultaneous study of multi-objective models, multi-period time horizons, conic constraints, discrete decisions, metaheuristics methods, and uncertainty. Cui et al. [5] believe that the time inconsistency of the portfolio optimization model has become an essential yet challenging research topic.

This article explores the impact of using a multi-period stock portfolio optimization model on portfolio selection and investment strategies. By considering uncertainty over multiple time periods, the model aims to make more informed investment decisions that positively influence the stock market. The study introduces new methods for optimizing portfolios, including cone constraints and stochastic decisions, to reduce costs. The goal is to develop a realistic framework that helps investors make profitable decisions, effectively manage assets, and succeed in the financial markets by considering various constraints and time horizons.

2 | Literature Background

A logical investor pays attention to the risk of the stock portfolio returns besides focusing on maximizing it. In Markowitz [1], the risk has been measured by the variance of historical returns. Thus, according to the mentioned cases, investors have made two goals to form a stock portfolio, turning the stock portfolio into a multi-objective issue. Moreover, uncertainty in issues makes it difficult to solve problems. Thus, Sharpe [6] proposed a new indicator to predict the stock return rate based on the computational issues of the Markowitz model. This indicator was called beta and measured the sensitivity of the stock return rate to alter in the stock market index. He suggested that the stock market index can clarify the economic situation of each nation. Researchers have utilized different techniques and conducted various studies to maximize stock portfolio returns. Ghahtarani and Najafi [7] used ideal planning to research the stock portfolio issue. They utilized a robust optimization approach to demonstrate the uncertainty of the parameters related to the portfolio model.

Similarly, the robust optimization models for both variants of Stable Tail-Adjusted Return Ratio (STARR). Deviation Mixed Conditional Value-at-Risk (DMCVaR) and Mixed Conditional Value-at-Risk (MCVaR) were introduced by Goel et al. [8]. They used copulas under joint ambiguity in the distribution. Both models were illustrated to be mathematically manageable linear programs. They applied a two-step procedure among the assets to capture the joint dependence structure. Firstly, they used the Autoregressive Moving Average Glosten Jagannathan Runkle Generalized Autoregressive Conditional Heteroscedastic (ARMA-GJR-GARCH) model to extract the filtered remaining from the return series of each asset. Secondly, they exploited the regular vine copulas to model the mutual dependence among the transformed residuals. The performance of the two models was compared with their standard equivalent when the joint distribution in the latter was described using a Gaussian copula only. Using the rolling window analysis, they examined the obtained portfolios' effectiveness compared to the multivariate GARCH and Markowitz models.

The relationship between the volatility of stock return and exchange rate uncertainty was investigated by Resekhi et al. [9]. To study the effect of exchange rate volatility on the return, they used CAPM and ICAPM, and a portfolio approach was employed to study the effect of asset return volatility on the exchange rate. The research hypotheses have been tested by using a multivariate GARCH model. The empirical findings showed the positive effect of actual exchange rate volatility on the stock return fluctuations. In contrast, the stock return fluctuations do not affect the exchange rate volatility.

Khandan [10] compared the performance of the mean or median optimization. He also incorporated various risk measures into models to test which performs better besides the median as an alternative to mean-variance models. Results indicated that the median performed better in portfolio optimization. The model of median maximization gained higher returns in seventy percent of cases and a higher average. In other words, a higher portfolio value would be obtained using the median in optimization. In comparison among different median optimization models applied it was shown that CVaR and MAD risk measures control the risk better than VaR and maximum loss and obtain even further diversification.

In another way, optimizing a fuzzy and multi-period portfolio problem with a minimum transaction, lots were considered by Liu and Zhang [11]. A mean-semi variance portfolio selection model was formulated in the proposed model based on possibility theory to minimize the cumulative risk and maximize the ultimate wealth over the entire investment horizon. The proposed model considered the risk, return, transaction costs, cardinality constraint, diversification, and minimum transaction costs. A fuzzy decision strategy was employed to convert the proposed model into a single objective mixed-integer nonlinear programming problem. Then, a genetic algorithm was used for the solution. For portfolio optimization, the portfolio assets were modeled through a hidden Markov model in a discrete period by Erlwein-Sayer [12], where the volatility and drift of the single assets could switch between various situations. They considered different parametrizations of the involved asset covariance and realized that strategies such as equal weights were proposed in many settings to outperform investment methods for simulated returns.

The multi-period mean-CVaR portfolio decision model is prone to time inconsistency problems that drive CVaR investors away from the pre-committed portfolio strategy Cui et al. [13].

Cui et al. [13] considered time-consistent and self-coordination strategies proposed to solve the time-inconsistent issue arising from other sequential decision problems. However, these strategies are rarely studied under the multi-period mean-CVaR portfolio decision framework. The revealed time-consistent strategy is a piecewise linear function of the wealth level. In contrast, the other parameters can be computed by solving a series of mixed-integer programming problems offline. The self-coordination strategy can be formulated as a convex program with a quadratic constraint. Cui et al. [13] also proved that the pre-committed and time-consistent strategies were the extreme cases of the self-coordination strategy.

Liechty and Saglam [14] studied the portfolio selection problem as a Bayesian decision problem. They compared the mean-variance–efficient skewness and traditional mean-variance portfolios. A bi-level programming problem was developed for risk by using observed weights in the market to explore the investors' preferences.

Similarly, Raei et al. [15] used criteria like kurtosis and skewness as new measurements and the Fama and French 3-factor model to compare their ability to determine the difference in stock returns. The results showed that market risk premium, company size, and kurtosis were more efficient than other factors like B/M and skewness in determining the risk of portfolios.

Pal et al. [16] introduced a strategy of portfolio formation through horizontal and vertical clustering. The clustering algorithm integrates stocks in the portfolio with exposure limits on each stock. Variable-length Non-dominated Sorting Genetic Algorithm (NSGA-II) was used to obtain the near Pareto optimal portfolios. Quarter-wise weights of each portfolio's constituent stocks were determined through the proposed single objective Genetic Algorithm based on the Markowitz model. The proposed model enabled dynamic reallocation of the portfolios and could integrate the macroeconomic environment of the time.

Corsaro et al. [17] investigated the problem of optimizing the long-term investment method so that the investor can find the opportunity to avoid severe loss and exit the investment before maturity. Their setting was a multi-period one. A model in the Markowitz context was developed based on a fused lasso strategy. In the context of portfolio management, Law et al. [18] differentiated the evaluation strategies for two significant investment methods, namely, active and passive, for managing the portfolio. Claiming that the tracking error did not exceed a certain threshold, they applied a non-inferiority test, which gets the null hypothesis when there is insufficient proof to reject it.

Peykani et al. [19] proposed an investment portfolio optimization in uncertain data. Accordingly, the main goal of this paper is to propose a novel Fuzzy Multi-Period Multi-Objective Portfolio Optimization (FMPMOPO) model that is capable of being used under data ambiguity and practical constraints, including budget constraint, cardinality constraint, and bound constraint.

Zhao et al. [20] focused on the multi-objective portfolio optimization problems considering the cardinality constraints. They proposed a multiple population's co-evolutionary particle swarm optimization algorithms based on multiple populations for a multiple objectives model with four advantages. A robust multi-period portfolio selection model was presented based on prospect theory, considering Moghadam et al.'s [21] interval semi-absolute deviation measure.

They considered the behavioral factors influencing investors' utility in the portfolio selection model for maximizing the utility value function of prospect theory. The transaction cost, the model diversification degree, and the cardinality constraint were utilized to improve the quality of the results. In another study that has been done by Yu et al. [22], a multi-period model was proposed, and variance and entropy were used for measuring the risk of portfolios. The results showed that the intelligent hybrid algorithm is superior to traditional methods. As a result, the hybrid GA with WNN could perform better for optimization models that have non-smooth functions.

The reviewed papers were summarized in *Table 1* for the research gap analysis and convincing the current research contribution.

Table 1. Literature background summary.

Authors	**Model/Methods**	**Objective Function**		**Investment Horizon**		**Solving Method**	
		Single Objective	**Multi-Objective**	**Single-Period**	**Multi-Period**	**Algorithm**	**Accurate**
Erlwein Sayer [12]	Hidden markov model in a discrete period	✓			✓	✓	
Goel et al. [8]	GARCH and markowitz model	✓		✓			✓
Law et al. [18]	Active and passive investment methods	✓		✓			✓
Pal et al. [16]	Horizontal and vertical clustering	✓		✓		✓	
Peykani et al. [19]	alpha-cut method		✓		✓		✓
Corsaro et al. [17]	Fused lasso strategy	✓			✓		✓
Zhao et al. [20]	Hybrid binary and return risk ratio heuristic strategy		✓	✓			✓
Abolmakarem et al. [23]	Deep-learning approach		✓		✓	✓	
Wang et al. [24]	Mean-CVaR model	✓		✓		✓	
Current research	Cone constraints, uncertain and stochastic discrete decisions		✓		✓	✓	

In the real world, stock returns should be analyzed using fuzzy, robust, or probabilistic planning according to many choices available in the capital markets since the return on capital during various periods cannot be considered steady. Very few simultaneous studies have been done on multi-objective models, multi-period time horizons, cone constraints, discrete decisions, metaheuristics methods, and uncertainty. Thus, this article aims to model a multi-objective and multi-period stock portfolio considering cone constraints and uncertain and stochastic discrete decisions.

3 | Methodology

The research framework is presented in *Fig. 1*.

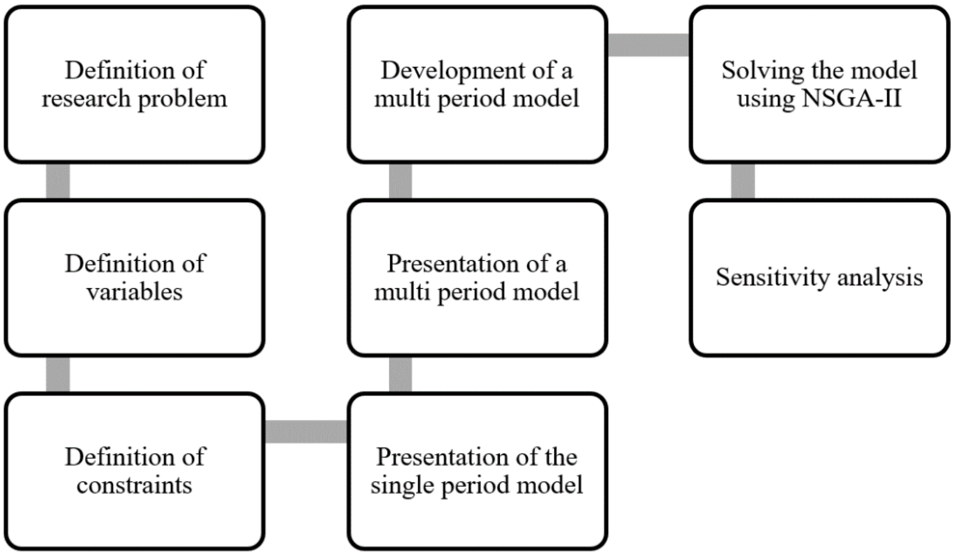

Fig. 1. The research framework.

A logical framework maximizes the expected return on a certain level of risk while making investment decisions; in this regard, Markowitz's single-period model is mentioned. Optimizing the one-period mean-variance could be generalized to multi-period planning. Thus, the portfolio could be invested by maximizing profits on the horizon, minimizing risk over discrete periods after the initial investment, and using the scenario tree to create a multi-period model. *Table 2* shows the nomenclature of the mathematical model.

Table 2. Description of mathematical notations of the model.

Notation	Description	Notation	Description
r	The vector of expected returns on each asset	l_{min}	The minimum number of stock exchange groups in the portfolio
w	The vector of initial holdings in each asset	C	A binary variable associated with each stock exchange group
x^+	Amounts bought in each asset	w_{min}	The minimum amount transacted in each asset
x^-	The vector of amounts sold in each asset	δ	A binary variable associated with each asset
$\sum$	The variance-covariance matrix of n assets	s	The short position threshold
Φ	The cumulative standard normal distribution function	Λ	Transaction cost matrix
W	At the end of the period wealth	∂_e	Variance of asset
w_{low}	At the end of the period threshold wealth level	c_b	The vector of buying transaction cost
s_{min}	The threshold investment level for each stock exchange group	c_s	The vector of selling transaction cost

The investor's objective is to choose the optimal trading methods for maximizing the end-of-period expected return. Indicating the expected return by $r \in R^{n+1}$ and the current portfolio holdings by $w \in R^{n+1}$, the expected total portfolio value at the end of the period is shown by [25]

$$W = \sum_{j=0}^{n} r_j \left(w_j + x_j^+ - x_j^-\right). \tag{1}$$

The selling and buying transactions involve transaction costs such as bid-ask spreads, commissions, brokerage, and taxes. Several strategies exist for modeling transaction costs, including convex and linear or

concave nonlinear cost functions. In this study, the quadratic convex transaction cost formulation is used. Thus, the total transaction costs are shown as a penalty term in the objective function:

$$\frac{1}{2}(x^+ + x^-)^T \wedge (x^+ + x^-). \tag{2}$$

Since $\wedge \in R^{(n+1).(n+1)}$ is the transaction cost matrix, which is obtained as a positive coefficient from the expected return covariance matrix, $\wedge$ is a definite and symmetric matrix.

The exact transaction cost is considered for both selling and buying transactions. However, it would be uncomplicated to include two quadratic terms in the objective function with various transaction cost matrices for every type of transaction.

A new variable, p, is introduced, so we rewrite the objective function as

$$\sum_{j=0}^{n} r_j \left(w_j + x_j^+ - x_j^-\right) - p. \tag{3}$$

$$\frac{1}{2}(x^+ + x^-)^T \wedge (x^+ + x^-) \leq 2p.$$

This constraint is equivalent to:

$$(x^+ + x^-)^T \wedge (x^+ + x^-) \leq (1 + p)^2 - (1 - p)^2. \tag{4}$$

Then, the last term was moved to the left-hand side, and both sides' square roots were taken. The second-order cone constraint is shown in the following term:

$$\left\| \begin{pmatrix} \wedge^{\frac{1}{2}} (x^+ + x^-) \\ 1 - p \end{pmatrix} \right\| \leq 1 + p. \tag{5}$$

$\wedge^{\frac{1}{2}}$ exists since $\wedge$ is positive definite [25].

As mentioned in the model, risk and return have been considered. The main concern is to maximize total expected returns with lower transaction costs in the objective function. Thus, we tend to limit the investor risk by using constraints. For the mentioned target, a conditional value at risk constraints is used. For each CVaR constraint K, it will be required that the expected return at the end of the period be above some threshold level W_k^{low} with a probability of at least n_k. Therefore,

$$W = \widehat{r^T}(w + x^+ - x^-). \tag{6}$$

Since $\hat{r}$ is the random vector of returns, it is required that:

$$p\left(W \geq W_k^{low}\right) \geq n_k. \tag{7}$$

It is assumed that the elements of r have joint Gaussian distribution so that W is normally distributed with a mean of:

$$r^T(w + x^+ - x^-). \tag{8}$$

Furthermore, the standard deviation is shown as below.

$$\left\| \sum^{\frac{1}{2}} \left(w + x^+ - x^-\right) \right\|. \tag{9}$$

$\sum$ is the covariance matrix of the returns. Thus, the CVaR constraints can be formulated as

$$p\left(\frac{W - r^T(w + x^+ - x^-)}{\left\| \sum^{\frac{1}{2}} \left(w + x^+ - x^-\right) \right\|} \geq \frac{W_{low}^k - r^T(w + x^+ - x^-)}{\left\| \sum^{\frac{1}{2}} \left(w + x^+ - x^-\right) \right\|}\right) \geq n_k. \tag{10}$$

As a result, considering Φ as a cumulative distribution function for each standard normal random variable, the second-order cone constraint CVAR is formulated as [25]

$$\Phi^{-1}(n_k)\left\|\sum^{\frac{1}{2}}\left(w+x^{+}-x^{-}\right)\right\| \le r^{T}(w+x^{+}-x^{-})-W_k^{low}. \tag{11}$$

Diversification is another powerful method to reduce risk in the portfolio.
Thus, diversification is imposed on the investor for sufficiently allocating large amounts in the different economic parts. Expressing this requirement, defining binary variables $C_k \in \{0,1\}$ for each stock exchange group k is done. If $C_k = 1$, the minimum amount of the portfolio allocation to the sector k is s_{min}. Otherwise, it is between the sum of short positions, long positions, and s_{min}. These requirements can be expressed with a constraint in the following form [26]:

$$-\sum_{j=1}^{n} s_j + (\sum_{j=1}^{n} s_j + s_{min})C_k \le \sum_{j\in s_k}(w_j + x_j^{+} - x_j^{-}) \le +\sum_{j=1}^{n} s_j + s_{min} + (1-(\sum_{j=1}^{n} s_j + s_{min}))C_k. \tag{12}$$

S_K is the set of assets that belong to stock exchange group k.

It also needs to introduce a cardinality constraint for expressing the diversification requirement [25].

$$\sum_{k=1}^{L} C_k \ge L_{min}. \tag{13}$$

Since transaction costs have been included in the model, the number of transactions will also be a concern. Therefore, a requirement can be imposed for investors only to hold a small active position. This requirement can be written using constraints of the following form by a new binary variable.

$$w_{min}\delta_j^{+} \le x_j^{+} \le \delta_j^{+}. \tag{14}$$
$$w_{min}\delta_j^{-} \le x_j^{-} \le \delta_j^{-}.$$

Since w_{min} is a predetermined proportion of the available capital and $x_j^{+}.x_j^{-} = 0$, the constraint can be reformulated in the following form [25].

$$w_{min}\delta_j \le x_j^{+} + x_j^{-} \le \delta_j. \tag{15}$$

The remaining constraints in this problem are placed in the general category of portfolio constraints.

$$\sum_{j=0}^{n}\left(w_j + x_j^{+} - x_j^{-}\right) = 1. \tag{16}$$

This constraint requires that 100% of the portfolio be allocated at the end of the investment period. Since we start with:

$$\sum_{j=0}^{n} w_j = 1. \tag{17}$$

This constraint can also be written in the following form.

$$\sum_{j=0}^{n} x_j^{+} = \sum_{j=0}^{n} x_j^{-}. \tag{18}$$

Constraint (18) provides a balance between the selling and buying transactions. Furthermore, another constraint that allows for short sales of the non-liquid assets by taking a limited short position for each one is defined below.

$$w_j + x_j^{+} - x_j^{-} \ge -s_j. \tag{19}$$

That s illustrates the short position limit for each non-liquid asset.

This one-period model is introduced, and the multi-objective multi-period model is introduced to complement the previous model. The creation of a scenario tree accomplishes this multi-period model.

The scenario tree makes discrete decisions at periods $t=1$ to T [26]. The existing nodes correspond to the leaves of the scenario tree. In the scenario tree, each node signifies an event.

$$e \equiv (s, t). \tag{20}$$

It means that scenario s occurs in period t.

Choosing the optimal trading strategies is the investor's objective function for maximizing the end-of-period expected return. The expected rate of return is denoted by r_e, and the current portfolio holdings are denoted by w_e [26]. Accordingly, the objective function z1 considers multi-period and rewriting the model with all the mentioned constraints.

It is also possible to evolve a single-objective model into a multi-objective model by adding a new objective function (z2), which leads to risk minimization. The research model and the constraints are introduced as follows:

$$\max z1 = p_e\, r_e w_{a(e)}.$$

$$\min z2 = \partial_e\, w_{a(e)}.$$

$$\text{s.t. } w_e = r_e w_{a(e)} + x_e^+(1-c_b) - x_e^-(1+c_s).$$

$$1^T x_e^+ = 1^T x_e^-.$$

$$1^T w_e \geq 0.90\left(1^T w_{a(e)}\right).$$

$$-\sum_{e\in N} s_e + \Big(\sum_{e\in N} s_e + s_{min}\Big)C_k \leq \sum_{e\in s_k} w_e \leq \sum_{e\in N} s_e + s_{min} + \Big(1-\Big(\sum_{e\in N} s_e + s_{min}\Big)\Big)C_k.$$

$$\sum_{k=1}^{L} C_{ke} \geq L_{min},$$

$$C \in \{0,1\},$$

$$w_{min}\delta_e \leq x_e^+ + x_e^- \leq \delta_e,$$

$$\delta \in \{0,1\},$$

$$x_e^+ \geq 0.$$

$$x_e^- \geq 0.$$

$$w_e \geq 0.$$

Minimizing the investment risk is the second objective function, where ∂_e indicates the variance of assets and $w_{a(e)}$ indicates the initial holdings in each asset related to an ancestor of event e. The parent node or ancestors of e in the scenario tree is denoted by a(e). The transaction cost is defined by w_e The current portfolio holdings for event e are balanced for both buying and selling strategies. The branching probability of an event is calculated by $p_e = \text{Prob}\ [e\backslash a(e)]$ where the probability of event e is computed by $P_e = \prod_{i=1}^{t} p(s,i)$ [26]. To impose risk constraints for the interior branches of the tree, we defined $1^T w_e \geq 0.90\left(1^T w_{a(e)}\right)$ as done by Saglam et al. [26], where $e= 1, \ldots$ T-1.

The total portfolio allocation is between the sum of those assets' short position and the sum of those assets long position and the threshold level s_{min}. S_k is the set of assets that belong to the stock exchange group k, $k=1, \ldots, L$, and $e \in N$, and N represents the set of all nodes in the scenario tree. In order to express the diversification requirement, a cardinality constraint was considered, as done by Saglam et al. [26]. x_e^+ and x_e^- are the variables associated with the buying and selling transactions, which must be non-negative.

The NSGA-II, was utilized to solve the model. This method was suggested for use in the previous study by Shiri Ghahi et al. [27] to compare portfolio optimization models in a fuzzy credibility environment. Similarly, end-of-period wealth maximization and risk minimization were aimed at in the mentioned study. The proposed model was solved using MATLAB software. The related code, written in MATLAB software, is presented in GitHub[1].

Due to the dual-purpose mode, an algorithm must solve problems in a two-objective situation. In the current study, using a two-objective genetic algorithm was preferred, considering the efficiency and performance of one of the most broadly utilized metaheuristics. First, the background of genetic algorithms is considered; afterward, the algorithm's operation process is presented.

The general scheme of the NSGA-II algorithm is illustrated in *Fig. 2*. There are two categories of answers from the beginning of the second cycle onwards, one a group of the parent population from the previous stage indicated by P_t, and the other group, the population of offspring resulting from the function of two crossover and mutation operators on parents indicated by Q_t. To start the next cycle, these two populations must be removed to maintain the initial population constant. Firstly, the members of $P_t \cup Q_t$ should be ranked. At this stage, discover those population members that are non-dominated and dedicate them to F_1 based on the non-dominated sorting. Afterward, for other members, non-dominated assorting was performed again by ignoring the impact of F_1 rank members of the population. At this stage, we mark members who have never been dominated with a rating of F_2, and this process will be continued until all member populations are ranked. On the F_3 front, some members are not in the selection area, but in terms of fronting, all members have a similar score. Thus, there are issues of crowding distance in the selection of members. The answers will be spread evenly and regularly on the Pareto front by calculating the crowding distance. As it can be seen, several P_{t+1} members of $P_t \cup Q_t$ should be selected based on their rank, and the rest should be eliminated.

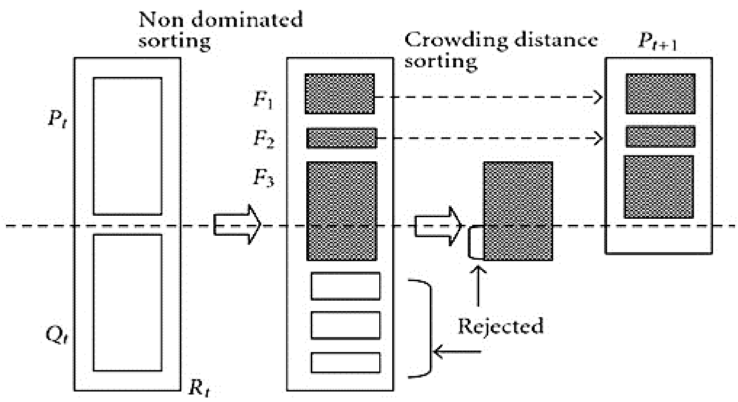

Fig. 2. NSGA-II Procedure [28].

Generally, the steps of the metaheuristics method are as follows:

Step 1. describe the initial data and algorithm parameters: in this step, fitness functions, the number of the initial population, crossover percentage, and mutation percentage are defined.

Step 2. production of the initial population: they are typically generated randomly by producing populations of chromosomes and consisting of answers to the problem.

Step 3. perform multi-objective operations (non-dominated sorting, calculating crowding distance, sorting).

Step 4. start of the main loop.

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[1] https://github.com/ZohrehEbrahimi16/Matlab-Code

Step 5. production of children by crossover method: in this step, two parents are selected to produce a new chromosome in one of the following ways. Children are created by distributing the parents' traits in them.

I. The roulette wheel: in this way, the more fitness element has a better chance of being selected.

II. Tournament: in this method, a small subset of chromosomes was randomly selected and competing and chosen according to the degree of fitness.

III. Random: in this method, a selection is made randomly from the existing responses.

IV. After selecting two parents, two children (two new answers) were created according to these two parents. There are four crossover methods at this stage, as follows:

One-point crossover: along two chromosomes chosen as parents, select a point randomly, and the chromosomes swapped from that point (*Fig. 3*).

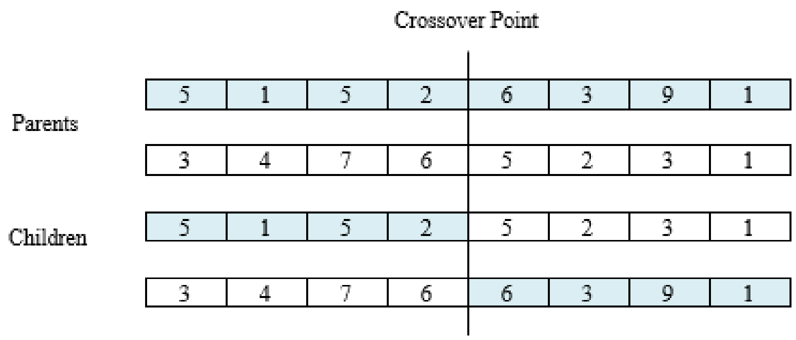

Fig. 3. One-point crossover.

Two-point crossover: along two chromosomes selected as parents, the values are displaced between these two points, and two points are randomly assigned (*Fig. 4*).

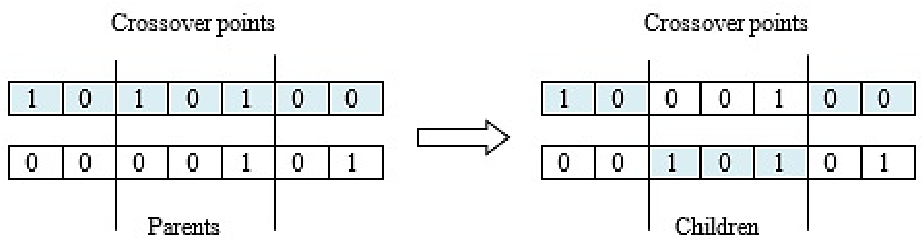

Fig. 4. Two-point crossover.

Mask crossover: a zero and one mask vector is randomly created to determine which parent's gene to choose for the composition and creation of offspring (*Fig. 5*).

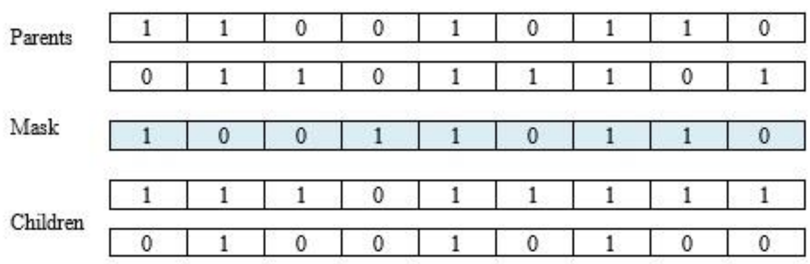

Fig. 5. Mask crossover.

Uniform-arithmetic crossover: in this method, a vector is randomly created between zero and one, and children are born from the following formula:

$$y_1 = x_1 * R + x_2 * (1 - R) = x_2 * R + x_1 * (1 - R) y_2. \quad (22)$$

Step 6. using mutation method for production of offspring: at this stage, firstly, a parent (answer) should be chosen in one of the methods that include roulette wheel, tournament, or random. One of the mutation methods is to create a child as follows after selecting the desired parent.

Swap: by choosing this method, two genes are randomly selected and displaced (*Fig. 6*).

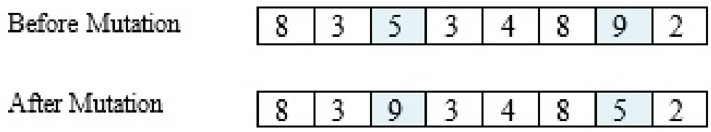

Fig. 6. Swap mutation.

Reversion: in this method, two genes are randomly chosen, and the genes between them are reversed (*Fig. 7*).

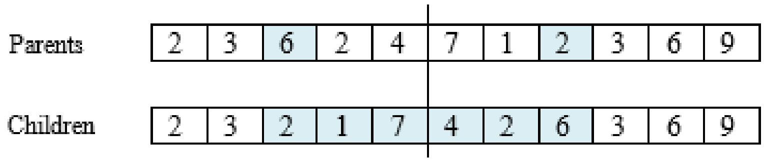

Fig. 7. Reversion mutation.

Uniform: In this method, several genes are selected (at least one and a maximum of ten or twenty percent of existing genes); afterward, a stochastic value is added or deducted from them (*Fig. 8*).

(1.31 4.54 3.73 2.81 6.36)
(1.31 4.54 3.62 2.90 6.36)

Fig. 8. Uniform mutation.

Step 7. Aggregating the previous and new generations: at this stage, the initial and children populations merge, creating a new population.

Step 8. Performing multi-objective operations (none dominated sorting, calculating crowding distance, sorting).

Step 9. The best selection: according to the necessary initial population, the best are selected at this stage.

Step 10. Performing multi-objective operations: (none dominated sorting, calculating crowding distance, sorting).

Step 11. Returning the starting point of the main loop for establishing termination condition: these conditions include: 1. The number of repetitions, 2. Time, 3. Lack of recovery, and 4. Convergence. In most cases, the number of repetitions is used. In *Fig. 9*, the steps of the Genetic algorithm are illustrated.

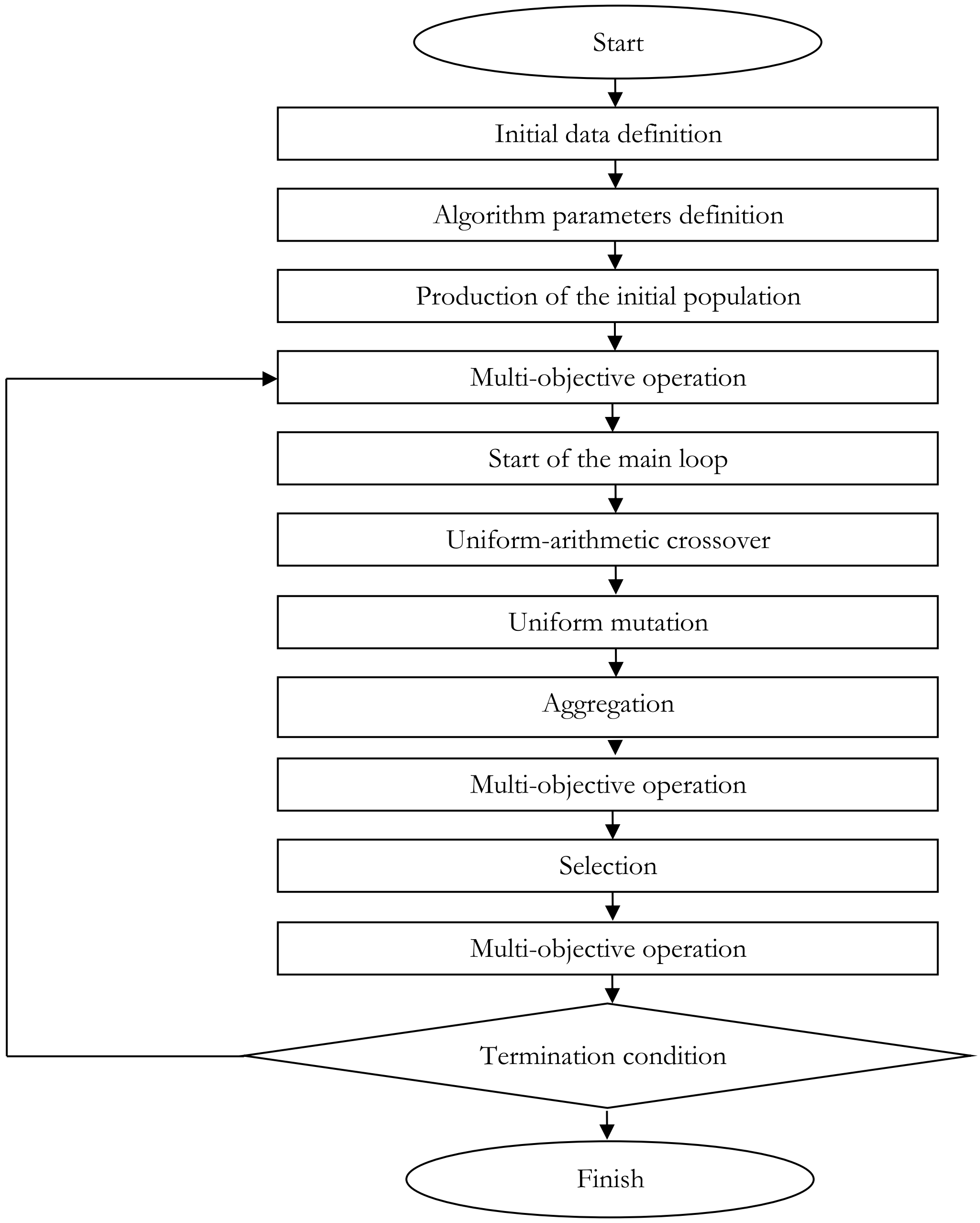

Fig. 9. The NSGA-II execution steps.

4 | Results and Discussion

In this section, the model solution results and sensitivity analysis of the proposed model are presented.

4.1 | Empirical Study

Ten of Iran's largest stock exchange groups include the Chemical Products Group, Basic Metals Group, Industrial Multidisciplinary Group, Investments Group, Petroleum Products Group, Group of banks and credit Institutions, Automobile and Parts Manufacturing Group, Pharmaceutical Materials and Products Group, and Auxiliary activities group to intermediate financial institutions[1]. As an empirical study, 14 assets,

[1] https://www.tse.ir/

X1 to X14, were randomly chosen from some stock exchange groups. *Table 3* lists these assets and their related stock exchange groups.

Table 3. The list of 14 assets chosen from various stock exchange groups.

k	Stock Exchange Group	Assets	k	Stock Exchange Group	Assets
1	Chemical products	X1, X2, X10	4	Products	X6, X14
2	Investments	X3, X12	5	Industrial multidisciplinary	X7, X8, X13
3	banks and credit institutions	X5, X9	6	Automobile and parts manufacturing	X4, X11

The scenario tree is constructed using monthly returns of the closing price of the stocks from June 2019 to July 2022. Due to the confidentiality of the relevant information, the names of the assets have not been given.

4.2 | Results/Pareto Diagram

The problem is solved using the second version of the multi-objective genetic algorithm by non-dominated sorting. The parameters of the problem are defined as shown in *Table 4*.

Table 4. Genetic algorithm parameters.

Mutation Rate	Mutation Percentage	Crossover Percentage	Initial Population	Number of Repetitions
0.02	0.4	0.7	50	100

Crossover percentage, mutation percentage, and mutation rate are achieved based on trial and error and are mainly chosen from zero to one. The problem was solved using MATLAB software coding based on the parameters and proposed steps. The results are provided in the form of *Fig. 10* and *Fig. 11*.

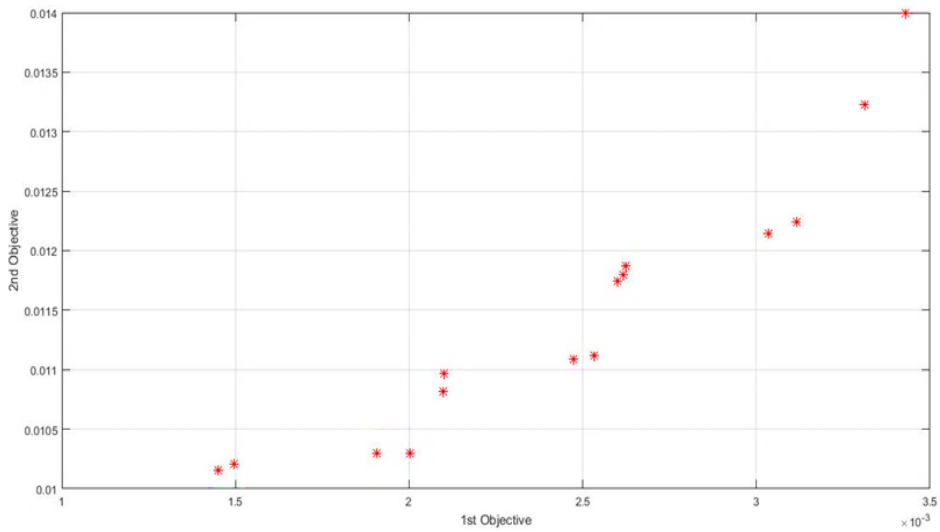

Fig. 10. Pareto chart.

Fig. 10 signifies the existence of two objective functions. The Pareto chart above shows the correlation between the first and second objective functions, the total return, and the entire risk.

In the Pareto chart, the horizontal axis specifies the expected return, and the vertical axis signifies the risk. All the obtained answer in the diagram has the same value. In other words, none was superior to the other, and they were selected only based on the level of utility of individuals. It also provides investment for risk-averse investors. As illustrated in *Fig. 10*, the higher the level of risk-taking, the higher the rate of return. Thus, according to the results, the possibility of selecting the optimal investment of the stock portfolio in financial markets is provided regarding the obtainable risk and expected return rate.

The optimal amounts of property allocated to the stock portfolio are shown as percentages in *Fig. 11*.

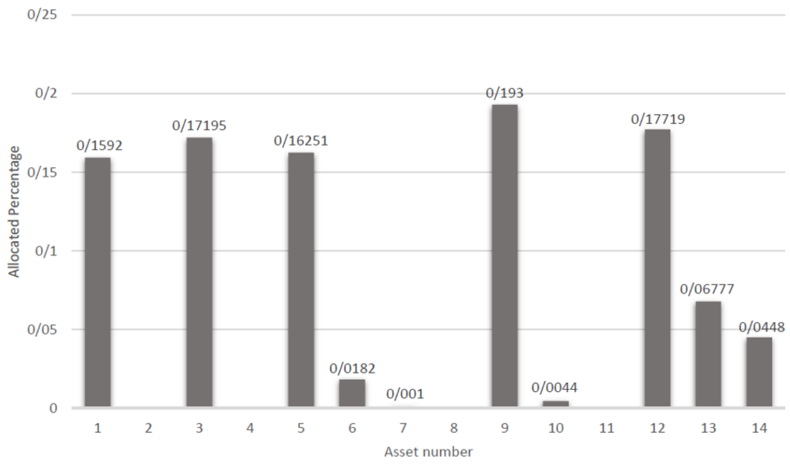

Fig. 11. The optimal amounts of wealth allocated to the stock portfolio.

This section illustrates the amount of wealth allocated per asset. As seen, assets 2, 4, 8, and 11 (X2, X4, X8, X11) account for zero percent, and asset 7 (X7) account for a tiny percentage. On the contrary, asset 9 (X9) has allocated about 20% of the capital. Other assets are also among the mentioned values.

Opportunities and threats are being considered so that investors can decide and select the optimal investment of the stock portfolio, following a review of the percentages assigned to existing assets. In other words, the best trading strategy can be achieved by selecting the existing opportunities and investing in stocks in which a more significant percentage of capital results in profitability. Conversely, not choosing assets is considered a threat, and assigning a low percentage or even zero to them signifies that they are unprofitable. In addition, the acquired percentages represent the prioritization of the profitability of the existing assets.

4.3 | Sensitivity Analysis

In the following, the sensitivity analysis of the model is performed based on increasing the values of the different parameters. Sensitivity analysis provides a base for investors to consider uncertainty in decision-making. In some cases, sensitivity analysis may not significantly influence the final output of the model, and, likely, the model is not sensitive to that constraint. On the contrary, a change in a constraint may modify the model output in general, which specifies the high sensitivity of the model to that constraint. Thus, sensitivity analysis plays a significant role in investor decision-making and prioritizing existing constraints.

The following are the results of sensitivity analysis in four scenarios.

4.3.1 | Increasing the minimum level invested in each stock exchange group

The first parameter is to increase the minimum level invested in each stock exchange group. As evident, slight changes in the capital allocation are achieved by increasing the parameter. The difference between allocations can be seen in the comparison section in *Fig. 12*.

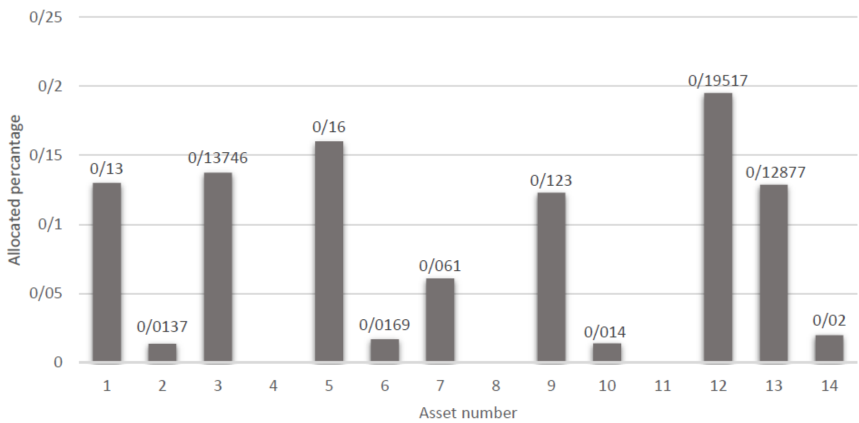

Fig. 12. Increasing the minimum level invested.

4.3.2 | Increasing the minimum number of stock exchange groups

The necessity of a minimum number of stock exchange groups in the stock portfolio is in the second parameter. In this part, increasing the mentioned parameter has led to more alteration in the allocation of assets; for instance, the highest asset is allocated to number 12, and asset 2 has a percentage greater than zero, as seen in *Fig. 13*.

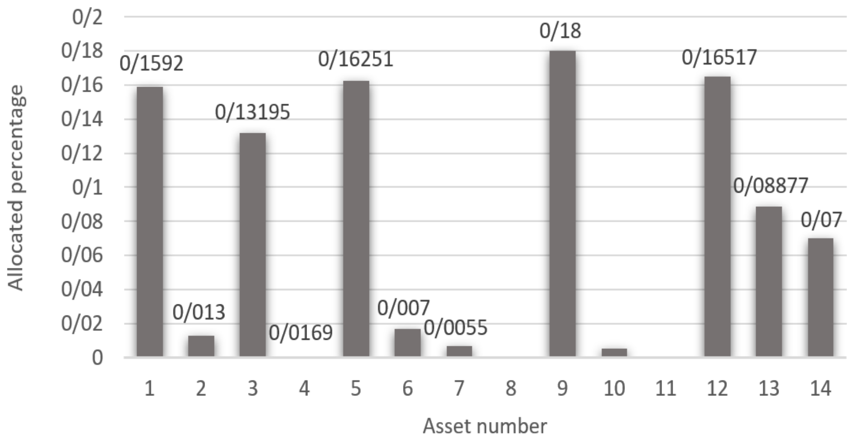

Fig. 13. Increasing the minimum number of stock exchange group.

4.3.3 | Increasing the wealth amount at the end of the period

The third parameter is increasing the amount of wealth at the end of the period. In this part, asset 12 allocated the most considerable capital as the most effective asset. Apart from this, asset 13 has a higher percentage. Assets 4, 8, and 11 have zero values, which had the same situation in the previous cases. Overall, the increase in wealth at the end of the period can substantially affect the amount of the allocated capital (*Fig. 14*).

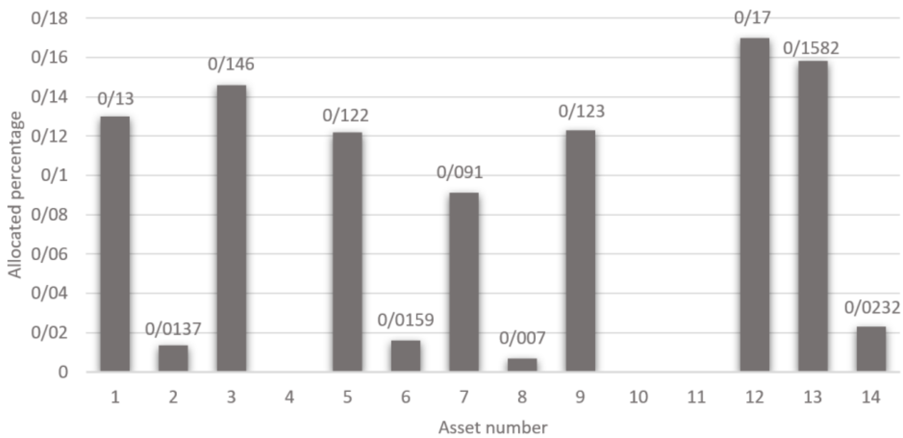

Fig. 14. Increasing the wealth amount at the end of the period.

4.3.4 | Increasing the minimum amount transacted in each asset

The minimum amount transacted in each asset, considered a base, was increased in scenario four. Accordingly, the above increase could impact the capital allocation. For instance, asset number 3 could attain more than 14% by increasing the minimum amount traded on each asset, which did not exist in the previous cases. In contrast, asset number 8 has assigned a value greater than zero. In addition, asset number 10 also obtained the value of zero, which specifies the impact of increasing the amount traded on each asset (*Fig. 15*).

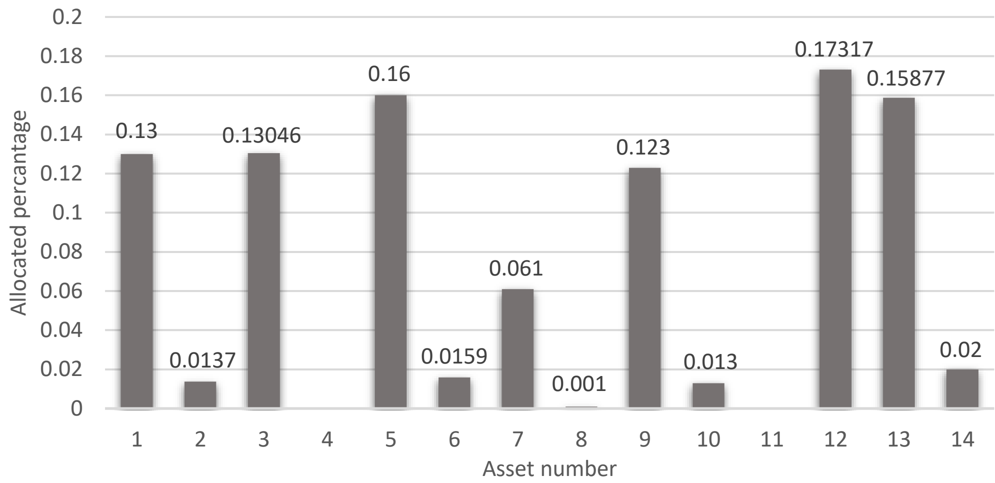

Fig. 15. Increasing the minimum amount transacted in each asset.

4.3.5 | Comparison of the results of different scenarios

The comparison of the results of the basic model against the results of the sensitivity analysis in the mentioned scenarios is shown in *Table 5*. As can be seen, the results obtained in each scenario can be considered.

In comparing various scenarios, asset numbers 4 and 11 have zero percent in all situations. Interestingly, asset numbers 4 and 11 were selected from the same stock exchange group. This indicates that in the studied period, this stock exchange group was not worth investing in any scenario. This analysis helps investors avoid investing in this group and avoid possible losses. In addition, asset number 9 was considered the most efficient asset in the primary result and increased the parameter of the minimum level invested in each stock exchange group. Afterward, asset number 12, with the increase of the following parameters, replaced it. A similar situation occurred with asset number 1, and in the primary result and the first scenario, an equivalent percentage has been acquired. However, this percentage decreased steadily in the other scenarios by increasing the other parameters.

Table 5. The results of the sensitivity analysis versus the basic model.

Asset	Basic Results	Increasing the Minimum Level Invested in Each Stock Exchange Group	Increasing the Minimum Number of Stock Exchange Groups	Increasing the Wealth Amount at the End of the Investment Period	Increasing the Minimum Amount Transacted in Each Asset
X1	0.1592	0.1592	0.13	0.13	0.13
X2	0	0.013	0.0137	0.0137	0.0137
X3	0.17195	0.13195	0.13746	0.13046	0.146
X4	0	0	0	0	0
X5	0.16251	0.16251	0.16	0.16	0.122
X6	0.0182	0.0169	0.0169	0.0159	0.0159
X7	0.001	0.007	0.061	0.061	0.091
X8	0	0	0	0.001	0.007
X9	0.193	0.18	0.123	0.123	0.123
X10	0.0044	0.0055	0.014	0.013	0
X11	0	0	0	0	0
X12	0.17719	0.16517	0.19517	0.17317	0.17
X13	0.06777	0.08877	0.12877	0.15877	0.1582

Regarding asset number 8, the value of zero was assigned to it in the initial state, and the first two parameters were increased; in the following parameters, this amount was raised by a small percentage. Regarding asset number 10, the allocation rate for this asset reached zero in increasing the parameter of the minimum amount traded in each asset. Such a situation did not exist in previous cases. In asset number 3, this asset was initially

about 17% of the capital, slightly altered in the following parameters. Finally, it allocated a higher rate in response to the last parameter. Asset number 2 did not primarily have a percentage, and the percentage of capital allocation to this asset increased slightly by increasing the parameters.

Overall, it is evident that substantial changes have occurred in capital allocation results with each sensitivity analysis, and this issue could be interesting. The assets have changed with each sensitivity analysis. This issue can be significant and appealing from a managerial point of view and could be very important in the investor's decisions. It was noticed in some assets that each time the sensitivity analysis is still allocated a value of zero, the investor can prevent potential losses by not entering such assets.

5 | Conclusion

Nowadays, the concept of a stock portfolio or selecting an asset is the primary concern of investors in the stock market, which is optimal for actual and legal profitability. Selecting the optimal stock portfolio influences investors by creating a portfolio that maximizes investor utility. The financial methods are considered to meet this requirement. Apart from this, the optimization of the stock portfolio is determined according to the amount of each asset in the stock portfolio, with the two conflicting goals of minimizing risk and maximizing returns. Thus, developing strategies in financial markets is significant because it helps investors and investment companies to make decisions. In this regard, in this research, a model was developed for optimizing the stock portfolio of multi-objective and multi-period by considering cone constraints uncertain and stochastic discrete decisions using a multi-objective genetic algorithm. In addition, the proposed dual-objective multi-period model was examined. The second version of the multi-objective genetic algorithm with non-dominated sorting was utilized to solve the problem. At first, the problem was solved in general dimensions. The Pareto chart includes non-dominated answers.

The various amounts of wealth allocated to the stock portfolio are shown in *Fig. 11*. Some assets like numbers 2, 4, 8, and 11 allocate zero percent in an optimal investment, which is considered a threat because these are unprofitable assets. On the other side, asset number 9 has allocated about 20% of the capital, and other assets are less than this value that can be chosen based on the investor's utility. In other words, according to opportunities and threats, investors decide and select the optimal investment of the stock portfolio, considering a review of the percentages assigned to the selected assets. All the answers acquired on the above diagram had the same value. They had no superiority over one another and were chosen based on investors' utility and degree of risk-taking. Afterward, the issue of sensitivity analysis, which is particularly significant, was performed for four vital parameters.

Overall, the sensitivity analysis illustrated that the model reacts to the increase of these parameters. Increasing each parameter can alter the capital allocation ratio per asset or even make some assets more than zero, with zero value. The prioritization of capital allocation for each asset changes with increasing parameters; this specifies the model response and decision variables. The results illustrated that slight changes could appear for capital allocated by increasing the minimum level invested in each stock exchange group; obviously, these changes will be insignificant. In the comparison section, the differentiation between the allocations can be seen.

Moreover, the necessity of the minimum number of stock exchange groups in the stock portfolio causes more changes in asset allocation. For instance, some assets have the highest percentage allocated, and some have a percentage greater than zero value. Apart from this, asset number 12 is the most efficient asset and has allocated the most significant amount of capital; assets 4 and 11 have zero value in all scenarios. Overall, factors including wealth at the end of the period and the minimum transacted amount on each asset parameter can significantly influence the amount of the allocated capital. It should be noted that in each sensitivity analysis, there were essential changes in capital allocation results, which are remarkable and could be considered in decisions related to capital allocation in the stock portfolio and preventing possible losses for investors. Allocating investments among different asset classes, namely shares issued by companies with different market capitalization, is a pivotal strategy to minimize risk and increase gains. A mix of investments,

from most aggressive to safest, will earn the total return over time. The percentage of the portfolio devoted to each depends on the investors' time frame and their risk tolerance.

Considering that this research is one of the new works presented in optimizing the stock portfolio, it can be expanded in various ways to improve the literature in this field. For instance, various risks and more objective functions can be utilized and examined in the obtained mathematical model. In addition, due to the complexity and impossibility of exact solutions for solving these issues, meta-heuristics are suggested.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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